

Effect of magnetic field on dilepton production in a hot plasma

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Noncentral collision of heavy ions can generate large magnetic field in its neighbourhood. We describe a method to calculate the effect of this field on the dilepton emission rate from the colliding region, when it reaches thermal equilibrium. It is calculated in the real time method of thermal field theory. We find that the rate is affected significantly only for lower momenta of dileptons.

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I. INTRODUCTION

An important probe into the dynamics of heavy ion collisions is the detection of dilepton production in the process. Accordingly this topic has been investigated in detail [1–6]. Here we study the change in the production rate due to the magnetic field, which is produced in non-central collision of individual events [7–15].

According to the present understanding, the two nuclei colliding at ultra relativistic energies appear as two sheets of Color Glass Condensate [16]. Very shortly after collision a strongly interacting Quark Gluon system, called Glasma, is formed, which is out of thermal equilibrium [17–20]. After thermalization it gives rise to the Quark Gluon plasma (QGP) phase. Finally it evolves into a hadron gas. Dileptons are produced in all these phases. In this work we address this production in the QGP phase.

The effect of magnetic field in different processes arises through the altered propagation of particles in this field. A non-perturbative, gauge covariant expression for the Dirac propagator in an external electromagnetic field was derived long ago by Schwinger in an elegant way, using a proper-time parameter [21]. It has since been rederived and applied to many processes [22–29]. In the present work we expand the exact propagator in powers of the magnetic field.

The dilepton production rate is given in terms of the (imaginary part of the) thermal two-point current correlation function. The latter involves the thermal propagator for quarks in the magnetic field. In contrast to the oft-used imaginary time formulation of thermal field theory [30–33], we shall use the real time formulation [34–36]. The advantage is that we do not have frequency sums for the propagators, but at the cost of dealing with 2×2 matrices for them in the intermediate stage of calculation. The matrices admit spectral representations, just like the vacuum propagators, which we shall use in calculating the thermal correlation function.

In Section II we write the dilepton rate formula and describe our method to evaluate it. In Section III we outline Schwinger’s construction of spinor propagator in magnetic field, leading to its spectral representation. In Section IV we then calculate the thermal two-point correlation function of currents, present in the rate formula. Finally Section V contains the numerical results and discussion.

II. FORMULATION

The transition amplitude (Fig 1) from an initial state I , composed of quarks and gluons to a final state F of similar composition, along with the emission of a dilepton $l(p, \sigma)$ and $\bar{l}(p', \sigma')$ of momenta p and p' and z -component of spin σ and σ' is

$$\langle F, l(p, \sigma), \bar{l}(p', \sigma') | S | I \rangle. \quad (\text{II.1})$$

Here the scattering matrix operator S is given by the interaction Lagrangian

$$\mathcal{L}_{\text{int}} = -e (j^\mu(x) + J^\mu(x)) A_\mu(x), \quad (\text{II.2})$$

of lepton and quark currents

$$j^\mu(x) = \bar{\psi}(x) \gamma^\mu \psi(x), \quad J^\mu(x) = \frac{2}{3} \bar{u}(x) \gamma^\mu u(x) - \frac{1}{3} \bar{d}(x) \gamma^\mu d(x), \quad (\text{II.3})$$

coupled to the electromagnetic field $A_\mu(x)$. We assume the initial state to be thermal and look for inclusive probability. Then if N is the dilepton emission rate per unit volume, we get, after some calculation [36]

$$\frac{d^4 N}{d^4 q} = \frac{\alpha^2}{6\pi^3 q^2} e^{-\beta q_0} (-g^{\mu\nu} M_{\mu\nu}^+), \quad (\text{II.4})$$

where $q = p + p'$ is the dilepton momentum and $M_{\mu\nu}^+(q)$ is a thermal two-point function of the quark current

$$M_{\mu\nu}^+(q) = \int d^4 x e^{iq \cdot x} \langle J_\mu(x) J_\nu(0) \rangle. \quad (\text{II.5})$$

Here the symbol $\langle O \rangle$ stands for ensemble average of the operator O at temperature $1/\beta$,

$$\langle O \rangle = \text{Tr}(e^{-\beta H} O) / \text{Tr} e^{-\beta H}. \quad (\text{II.6})$$

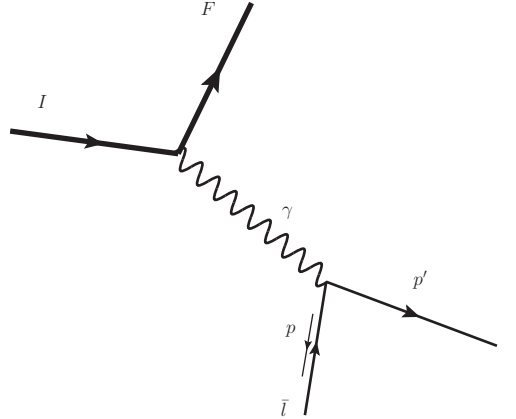


FIG. 1: Dilepton production amplitude in QGP phase. The states I and F consist of quarks and gluons, while $l\bar{l}$ is a dilepton. The weavy line corresponds to photon.

We briefly review how $M_{\mu\nu}^+(q)$ may be obtained in the real time thermal field theory [36]. We start with the time contour of Fig. 2 and define the time-ordered two-point function $M_{\mu\nu}(x, x')$ as

$$M_{\mu\nu}(x, x') = \Theta_c(\tau - \tau') i \langle J_\mu(x) J_\nu(x') \rangle + \Theta_c(\tau' - \tau) i \langle J_\nu(x') J_\mu(x) \rangle. \quad (\text{II.7})$$

where $x = (\tau, \vec{x})$, $x' = (\tau', \vec{x}')$ with the 'times' τ and τ' on the contour shown in Fig 2. The subscript c on the Θ -functions refers to contour ordering. Beginning with the spatial Fourier

transform, one can show that the vertical segments of the time contour does not contribute. Then the two-point function may be put in the form of a 2×2 matrix, which can be diagonalized with essentially one diagonal element

$$\overline{M}_{\mu\nu}(q) = \int_{-\infty}^{+\infty} \frac{dq'_0}{2\pi} \frac{\rho_{\mu\nu}(q'_0, \vec{q})}{q'_0 - q_0 - i\eta\epsilon(q_0)} \quad (\text{II.8})$$

where $\rho_{\mu\nu}(q)$ is the spectral function

$$\rho_{\mu\nu}(q) = \int d^4x e^{iq \cdot x} \langle [J_\mu(x), J_\nu(0)] \rangle \equiv M^+(q) - M^-(q) \quad (\text{II.9})$$

Eq. (II.8) gives us

$$\rho_{\mu\nu}(q) = 2 \text{Im} \overline{M}_{\mu\nu}(q). \quad (\text{II.10})$$

From the cyclicity of the thermal trace, we get the Kubo-Martin-Schwinger relation

$$M^+(q) = e^{\beta q_0} M^-(q) \quad (\text{II.11})$$

From Eqs. (II.9-11) we get

$$M^+(q) = \frac{2e^{\beta q_0}}{e^{\beta q_0} - 1} \text{Im} \overline{M}_{\mu\nu}(q) \quad (\text{II.12})$$

giving the dilepton rate (II.4) as

$$\frac{d^4 N}{d^4 q} = \frac{\alpha^2}{3\pi^3 q^2} \frac{W}{e^{\beta q_0} - 1}, \quad W = -g^{\mu\nu} \text{Im} \overline{M}_{\mu\nu}. \quad (\text{II.13})$$

Using the matrix which diagonalizes the 2×2 correlation matrix, we can relate the imaginary part of any one component, say the 11, of the correlation function to that of its diagonal element,

$$\text{Im} \overline{M}_{\mu\nu} = \epsilon(q_0) \tanh(\beta q_0) \text{Im}(M_{\mu\nu})_{11}. \quad (\text{II.14})$$

So far we utilize general properties of two-point functions to relate the problem to $(M_{\mu\nu})_{11}$. Taking τ and τ' on the real axis, the contour form (II.7) gives it as

$$M_{\mu\nu}(x, x')_{11} = i \langle T J_\mu(\vec{x}, t) J_\nu(\vec{x}', t') \rangle, \quad (\text{II.15})$$

where T as usual time orders the operators. It is this quantity which we have to calculate. To leading order in strong interactions, it involves only the thermal quark propagator. The magnetic field enters the problem through this propagator, which we find in the next section.

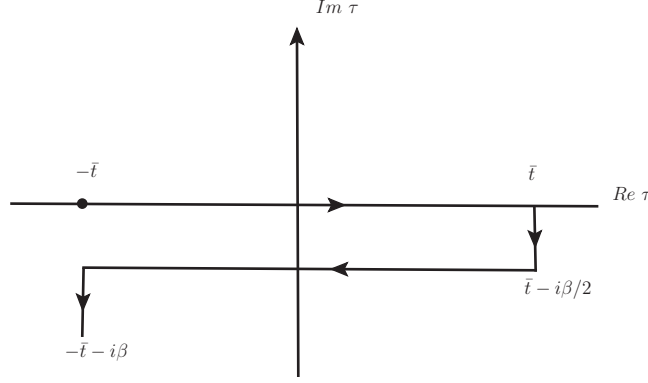


FIG. 2: The time contour in complex τ plane with $\bar{t} \rightarrow \infty$.

III. DIRAC PROPAGATOR IN MAGNETIC FIELD

In deriving the quark propagator we assume both u and d quarks to have the same absolute electric charge as that of the lepton. (The necessary correction will be included in our formulae at the end of Section IV). The Dirac Lagrangian in an external electromagnetic field

$$\mathcal{L} = \bar{\psi} [i\gamma^\mu (\partial_\mu + ieA_\mu) - m] \psi \quad (\text{III.1})$$

gives the equation of motion

$$[i\gamma^\mu (\partial_\mu + ieA_\mu) - m] \psi = 0. \quad (\text{III.2})$$

Then the propagator

$$S(x, x') = i\langle 0|T\psi(x)\bar{\psi}(x')|0\rangle \quad (\text{III.3})$$

satisfies

$$[i\gamma^\mu (\partial_\mu + ieA_\mu) - m] S(x, x') = -\delta^4(x - x'). \quad (\text{III.4})$$

Here $|0\rangle$ is the vacuum state of the Dirac field (in presence of A_μ). Defining states labeled by space-time coordinate (suppressing spinor indices), we regard $S(x, x')$ as the matrix element of an operator S

$$S(x, x') = \langle x|S|x'\rangle. \quad (\text{III.5})$$

Then Eq.(III.4) can be written as

$$(\gamma^\mu \pi_\mu - m)S = -1, \quad \pi_\mu = p_\mu - eA_\mu, \quad p_\mu = i\partial_\mu \quad (\text{III.6})$$

which has the formal solution ¹

$$S = \frac{1}{-\not{\pi} + m} = (\not{\pi} + m) \frac{1}{-\not{\pi}^2 + m^2}. \quad (\text{III.7})$$

¹ Another equivalent form follows by writing $(\not{\pi} + m)$ on the right in Eq.(III.7) [21], but we shall not use it.

Schwinger relates these quantities to the dynamical properties of a 'particle' with coordinate x^μ and canonical and kinematical momenta p^μ and π^μ respectively. Using their commutation relations, we get $\not{x}^2 = \pi^2 - \frac{e}{2}\sigma F$. Defining $H = -\pi^2 + m^2 + \frac{e}{2}\sigma F$ we can write Eq.(III.7) as

$$S = (\not{x} + m)i \int_0^\infty ds U(s), \quad U = e^{-iHs}. \quad (\text{III.8})$$

As the notation suggests, $U(s)$ may be regarded as the evolution operator of the particle with Hamiltonian H in time s .

We now go to Heisenberg representation, where the operators x_μ and Π_μ as well as the base ket become time dependent,

$$x_\mu(s) = U^\dagger(s)x_\mu U(s), \quad \Pi_\mu(s) = U^\dagger(s)\Pi_\mu U(s), \quad |x'; s\rangle = U^\dagger(s)|x'; 0\rangle \quad (\text{III.9})$$

Then the construction of the propagator reduces to the evaluation of

$$\langle x''|U(s)|x'\rangle = \langle x''; s|x'; 0\rangle, \quad (\text{III.10})$$

which is the transformation function for a state, in which the operator $x_\mu(s=0)$ has the value of x'_μ , to a state, in which $x_\mu(s)$ has the value x''_μ .

The equation of motion for $x_\mu(s)$ and $\pi_\mu(s)$ following from Eq.(III.9) can be solved to get

$$\pi(s) = -\frac{1}{2}eF e^{-eFs} \sinh^{-1}(eFs) (x(s) - x(0)) \quad (\text{III.11})$$

which may also be put in the reverse order on using the antisymmetry of $F_{\mu\nu}$. The matrix element $\langle x''; s|\pi^2(s)|x'; 0\rangle$ can now be obtained by using the commutator $[x_\mu(s), x_\nu(0)]$ to reorder the operators $x_\mu(0)$ and $x_\nu(s)$. We then get

$$\langle x''; s|H(x(s), \pi(s))|x'; 0\rangle = f(x''; x'; s)\langle x''; s|x'; 0\rangle \quad (\text{III.12})$$

where

$$f = (x'' - x')K(x'' - x') - \frac{i}{2}\text{Tr}[eF \coth(eFs)] - m^2 - \frac{e}{2}\sigma F, \quad K = \frac{(eF)^2}{4} \sinh^{-2}(eFs). \quad (\text{III.13})$$

We are now in a position to find the transformation function, which from Eq. (III.10) is found to satisfy

$$i \frac{d}{ds} \langle x''; s|x'; 0\rangle = \langle x''; s|H|x'; s\rangle \quad (\text{III.14})$$

It can be solved as

$$\begin{aligned} \langle x''; s|x'; 0\rangle &= \phi(x'', x') \cdot \frac{i}{(4\pi)^2 s^2} e^{-L(s)} \times \\ &\exp\left(-\frac{i}{4}(x'' - x')eF \coth(eFs)(x'' - x')\right) \exp\left(-i(m^2 + \frac{1}{2}e\sigma F)\right) \end{aligned} \quad (\text{III.15})$$

where

$$L(s) = \frac{1}{2} \text{Tr} \ln [(eFs)^{-1} \sinh(eFs)] . \quad (\text{III.16})$$

Here $\phi(x'', x')$ is a phase factor involving an integral over the potential A_μ on a straight line connecting x' and x'' . It will cancel out in our calculation. The spinor propagator is now given by

$$\begin{aligned} S(x'', x') &= i \int_0^\infty ds \langle x'' | (\not{x} + m) U(s) | x' \rangle \\ &= i \int_0^\infty ds [\gamma^\mu \langle x''; s | \pi_\mu(s) | x'; 0 \rangle + m \langle x''; s | x'; 0 \rangle] \end{aligned} \quad (\text{III.17})$$

with $\pi_\mu(s)$ and $\langle x''; s | x'; 0 \rangle$ given by Eqs. (III.11) and (III.15).

We now specialize the external electromagnetic field to magnetic field B in the z direction, $F^{12} = -F^{21} = B$. It is convenient to diagonalize the antisymmetric 2×2 matrix F^{ij} with eigenvalues $\pm iB$. Going over to spatial metric we get ²

$$\begin{aligned} S(x) &= \frac{i}{(4\pi)^2} \int \frac{ds}{s} \frac{eB}{\sin(eBs)} \exp \left[\frac{i}{4} x_\perp^2 eB \cot(eBs) - \frac{i}{4s^2} x_\parallel^2 - i(m^2 + \frac{1}{2} e\sigma F)s \right] \times \\ &\quad \left[\left(\frac{1}{2s} (x \cdot \gamma)_\parallel + m \right) (\cos \phi - \gamma^1 \gamma^2 \sin \phi) - \frac{eB}{2 \sin \phi} (x \cdot \gamma)_\perp \right] \end{aligned} \quad (\text{III.18})$$

which can be Fourier transformed to

$$\begin{aligned} S(p) &= i \int_0^\infty ds e^{is(p^2 - m^2 + i\epsilon)} e^{-isp_\perp^2 \left(\frac{\tan(eBs)}{eBs} - 1 \right)} \times \\ &\quad \left[(\not{p}_\parallel + m) (1 - \gamma^1 \gamma^2 \tan(eBs)) - \not{p}_\perp (1 + \tan^2(eBs)) \right]. \end{aligned} \quad (\text{III.19})$$

Expanding the exponential and tangent functions, we immediately get $S(p)$ as a series in powers of eB . To order $(eB)^2$ it is

$$S(p) = \frac{-(\not{p} + m)}{p^2 - m^2 + i\eta} + eB \frac{i(\not{p}_\parallel + m) \gamma^1 \gamma^2}{(p^2 - m^2)^2} - (eB)^2 \left[\frac{2\not{p}_\perp}{(p^2 - m^2)^3} - \frac{2p_\perp^2 (\not{p} + m)}{(p^2 - m^2)^4} \right]. \quad (\text{III.20})$$

To put the propagator (III.20) in the form of a spectral representation, we introduce a variable mass m_1 to replace $1/(p^2 - m^2)$ by $1/(p^2 - m_1^2)$, keeping the physical mass m unaltered at other places. The higher powers of the scalar propagator can then be expressed as derivatives of the propagator with respect to m_1^2 . We thus get

$$S(p) = -F(p, m, m_1) \frac{1}{p^2 - m_1^2} \bigg|_{m_1=m}, \quad (\text{III.21})$$

where

$$F = (\not{p} + m) + a i (\not{p}_\parallel + m) \gamma^1 \gamma^2 + b \not{p}_\perp + c p_\perp^2 (\not{p} + m) \quad (\text{III.22})$$

² For any two vectors a^μ and b^μ , we write $(ab)_\parallel = a^0 b^0 - a^3 b^3$ and $(ab)_\perp = a^1 b^1 + a^2 b^2$. Note that the longitudinal and transverse directions are defined with respect to the direction of magnetic field, not the collision axis of ions.

with coefficients a, b and c carrying the derivative operators,

$$a = -eB \frac{\partial}{\partial m_1^2}; \quad b = (eB)^2 \frac{\partial^2}{\partial (m_1^2)^2}; \quad c = -\frac{1}{3}(eB)^2 \frac{\partial^3}{\partial (m_1^2)^3}. \quad (\text{III.23})$$

From Eq.(III.21) the spectral function for $S(p)$ will be recognized as ³

$$\sigma(p) = F(p, m, m_1) \rho(p, m_1) \Big|_{m_1=m} \quad (\text{III.24})$$

with ρ being the spectral function for the scalar propagator of mass m_1

$$\rho(p, m_1) = 2\pi \epsilon(p_0) \delta(p^2 - m_1^2). \quad (\text{III.25})$$

The desired spectral representation for the spinor propagator in vacuum (in presence of magnetic field) can be written in the form

$$S(p) = \int_{-\infty}^{+\infty} \frac{dp'_0}{2\pi} \frac{\sigma(p'_0, \vec{p})}{p'_0 - p_0 - i\eta \epsilon(p_0)}, \quad (\text{III.26})$$

as can be readily verified by doing the p'_0 integral.

IV. THERMAL CURRENT CORRELATION FUNCTION

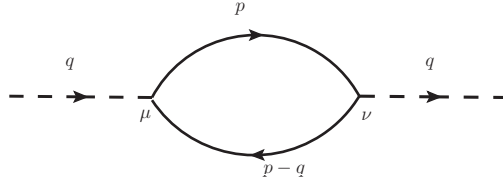


FIG. 3: Current correlation function to one-loop. The dashed and solid lines represent currents and quarks.

Like the thermal correlation function of currents, the thermal quark propagator can also be analyzed in the same way. In particular, its 2×2 matrix form can be diagonalized, again with essentially a single diagonal element, which turns out to be the vacuum propagator (in magnetic field) derived above. But in our calculation below, we need the 11– element of the original matrix, which is conveniently written as [36],

$$S_{11}(p) = \int_{-\infty}^{+\infty} \frac{dp'_0}{2\pi} \sigma(p'_0, \vec{p}) \left\{ \frac{1 - \tilde{f}(p'_0)}{p'_0 - p_0 - i\eta} + \frac{\tilde{f}(p'_0)}{p'_0 - p_0 + i\eta} \right\}, \quad \tilde{f}(p_0) = \frac{1}{e^{\beta p_0} + 1} \quad (\text{IV.1})$$

where the spectral function σ is given by (III.24).

³ We have three different spectral functions in this problem. The $\rho_{\mu\nu}$ introduced in Section II is the spectral function of current correlation function, while ρ and σ are spectral functions for the scalar and Dirac propagators.

The graph of Fig. 3 gives two terms involving u and d quark propagators. Assuming these to be equal (which is true only for $eB = 0$)⁴, we combine them to give

$$(M_{\mu\nu}(q))_{11} = \frac{5i}{3} \int \frac{d^4p}{(2\pi)^4} \text{Tr} [S_{11}(p)\gamma_\nu S_{11}(p-q)\gamma_\mu] \quad (\text{IV.2})$$

where the prefactor includes a factor of 3 for color of the quarks. Inserting the propagator (IV.1) in it, we want to work out the p_0 integral. For this purpose we write it as

$$M_{\mu\nu}(q)_{11} = \int \frac{d^3p}{(2\pi)^3} \int \frac{dp'_0}{2\pi} \rho(p'_0, \vec{p}) \int \frac{dp''_0}{2\pi} \rho(p''_0, \vec{p} - \vec{q}) K_{\mu\nu}(q) \quad (\text{IV.3})$$

where

$$K_{\mu\nu}(q) = i \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} N_{\mu\nu}(q) \left(\frac{1 - \tilde{f}'}{p'_0 - p_0 - i\eta} + \frac{\tilde{f}'}{p'_0 - p_0 + i\eta} \right) \times \left(\frac{1 - \tilde{f}''}{p''_0 - (p_0 - q_0) - i\eta} + \frac{\tilde{f}''}{p''_0 - (p_0 - q_0) + i\eta} \right) \quad (\text{IV.4})$$

with $\tilde{f}' = \tilde{f}(p'_0)$, $\tilde{f}'' = \tilde{f}(p''_0)$ and

$$N_{\mu\nu}(q) = \frac{5}{3} \text{tr} \left\{ \overleftarrow{F}(p, m, m_1) \gamma_\nu \overleftarrow{F}(p - q, m, m_2) \gamma_\mu \right\}. \quad (\text{IV.5})$$

Here the masses m_1 and m_2 are variables on which the mass derivatives act in the two propagators. The left arrow on F indicates the derivatives in it to be put farthest to the left (outside the integrals). As we are interested in the imaginary part of $K_{\mu\nu}$, we can put $p_0 = p'_0$, $p_0 - q_0 = p''_0$ and bring $N_{\mu\nu}$ outside the p_0 integral. Then it is simple to evaluate $K_{\mu\nu}$, from which we get its imaginary part. Extracting a factor $\coth(\beta q_0)$, it becomes linear in \tilde{f}' and \tilde{f}'' ,

$$\text{Im} K_{\mu\nu}(q) = N_{\mu\nu}(p'_0, p''_0) \pi (\tilde{f}'' - \tilde{f}') \coth(\beta q_0) \delta(p''_0 - p'_0 + q_0) \quad (\text{IV.6})$$

(The hyperbolic function will cancel out in (II.14)). Next, the p'_0 and p''_0 integrals in Eq.(IV.3) can be removed, using the delta functions present in the spectral functions, namely $\delta(p'_0 \pm \omega_1)$ and $\delta(p''_0 \pm \omega_2)$ with $\omega_1 = \sqrt{|\vec{p}|^2 + m_1^2}$ and $\omega_2 = \sqrt{|\vec{p} - \vec{q}|^2 + m_2^2}$. We need only the imaginary part in the physical region, $q_0 > (\omega_1 + \omega_2)$. From (II.14), (IV.3) and (IV.6), we then get

$$W = \pi \int \frac{d^3p}{(2\pi)^3} \frac{N_\mu^\mu(\omega_1, -\omega_2)}{4\omega_1\omega_2} \{1 - \tilde{n}(\omega_1) - \tilde{n}(\omega_2)\} \delta(q_0 - \omega_1 - \omega_2) \quad (\text{IV.7})$$

where we convert \tilde{f} 's to distribution functions, $\tilde{n}(\omega) = 1/(e^{\beta\omega} + 1)$.

Working out the trace over γ matrices in N_μ^μ we get

$$N_\mu^\mu = -\frac{40}{3} \left[(1 - a_1 a_2) p \cdot (p - q) - (b_1 + b_2 + a_1 a_2) [p \cdot (p - q)]_\perp + p \cdot (p - q) \{c_1 p_\perp^2 + c_2 (p - q)_\perp^2\} \right]. \quad (\text{IV.8})$$

⁴ For $eB \neq 0$, we include the necessary correction at the end of this section.

Let us now consider collision events in which the transverse components of momenta are small compared to the longitudinal ones, when we can omit the last two terms and calculate the dilepton rate analytically. Neglecting quark mass, we thus get

$$W = \frac{20\pi}{3} q^2 (1 - a_1 a_2) J \quad (\text{IV.9})$$

where

$$J = \int \frac{d^3 p}{(2\pi)^3 4\omega_1 \omega_2} \{1 - \tilde{n}(\omega_1) - \tilde{n}(\omega_2)\} \delta(q_0 - \omega_1 - \omega_2). \quad (\text{IV.10})$$

After working out this integral analytically, we shall apply the mass derivatives contained in a_1 and a_2 .

If θ is the angle between \vec{q} and \vec{p} , we can carry out the θ integral by the delta function in Eq.(IV.10). However a constraint remains to ensure that $\cos \theta$ remains in the physical region, as we integrate over the angle. We get

$$J = \frac{1}{16\pi^2 |\vec{q}|} \int d\omega_1 \Theta(1 - |\cos \theta|) \{1 - \tilde{n}(\omega_1) - \tilde{n}(\omega_2)\}. \quad (\text{IV.11})$$

The Θ -function constraint gives a quadratic expression in ω_1 ,

$$(\omega_1 - \omega_+)(\omega_1 - \omega_-) \leq 0 \quad (\text{IV.12})$$

where

$$\omega_{\pm} = \frac{q_0 R \pm |\vec{q}| \sqrt{R^2 - 4q^2 m_1^2}}{2q^2}, \quad R = q^2 + m_1^2 - m_2^2, \quad (\text{IV.13})$$

With the corresponding limits on ω_1 , we get [37]

$$\begin{aligned} J &= \frac{1}{16\pi^2 |\vec{q}|} \int_{\omega_-}^{\omega_+} d\omega_1 \left(1 - \frac{1}{e^{\beta\omega_1} + 1} - \frac{1}{e^{\beta(q_0 - \omega_1)} + 1} \right) \\ &= \frac{1}{16\pi^2 |\vec{q}| \beta} \left[\ln \left(\frac{\cosh(\beta\omega_+/2)}{\cosh(\beta\omega_-/2)} \right) - \ln \left(\frac{\cosh(\beta(q_0 - \omega_+)/2)}{\cosh(\beta(q_0 - \omega_-)/2)} \right) \right]. \end{aligned} \quad (\text{IV.14})$$

We now recall that the u and d quark charges were included correctly only in the currents but not in the propagators. The resulting correction will effect only e^2 , contained in a_1, a_2 in the expression for W . We can readily find that we need to multiply e^2 by 17/45 to restore the actual charges of the quarks in their propagators. Carrying out the mass derivatives in Eq.(IV.9) and going to the limit of zero quark masses, we finally get

$$W = \frac{5q^2}{12\pi |\vec{q}| \beta} \left[2 \ln \left(\frac{\cosh \alpha_+}{\cosh \alpha_-} \right) - \frac{17}{45} (eB)^2 \mathcal{M} \right], \quad (\text{IV.15})$$

where \mathcal{M} gives the effect of magnetic field to the leading order result,

$$\mathcal{M} = \frac{\beta^2}{8q^2} (\text{sech}^2 \alpha_+ - \text{sech}^2 \alpha_-) + \frac{\beta |\vec{q}|}{q^4} (\tanh \alpha_+ + \tanh \alpha_-). \quad (\text{IV.16})$$

Here we use the abbreviation $\alpha_{\pm} = \beta(q_0 \pm |\vec{q}|)/4$. Note that W is finite as $|\vec{q}| \rightarrow 0$.

V. NUMERICAL RESULTS AND DISCUSSION

Some earlier works estimate the magnetic contribution to dilepton rate in QGP phase in heavy ion collisions. Ref. [11] uses Weizsäcker-Williams equivalent photon approximation, in which the two vertices of Fig.1 become independent amplitudes involving the photon, whose probabilities are calculated in the magnetic field. In Ref. [14] the quark propagator is calculated using the method of eigenfunction expansion [22]. Here the anisotropy induced by the (constant) direction of the magnetic field is investigated in detail. In Ref. [15] the result for very high magnetic field is reported, taking the lowest Landau level into account.

Here we propose a different method to include the effect of magnetic field on the dilepton production rate. Assuming thermal equilibrium in the *QGP* phase, there results the correlation function of quark currents. This is evaluated with the quark propagator in magnetic field after expanding it up to $(eB)^2$. The calculation is carried out in the real time method of thermal field theory.

The plots of \mathcal{M} , the coefficient of $(eB)^2$ in W , as functions of invariant dilepton mass $m_{l\bar{l}} = \sqrt{q^2}$ and temperature T are shown in Fig. 4 for typical values of parameters. If the second order term in (IV.16) provides any indication of the behavior of the series, the expansion parameters are eB/q^2 and eB/T^2 . In Fig. 5 we plot $W/W_{B=0}$ as a function of $m_{l\bar{l}}$ for a few values of eB .

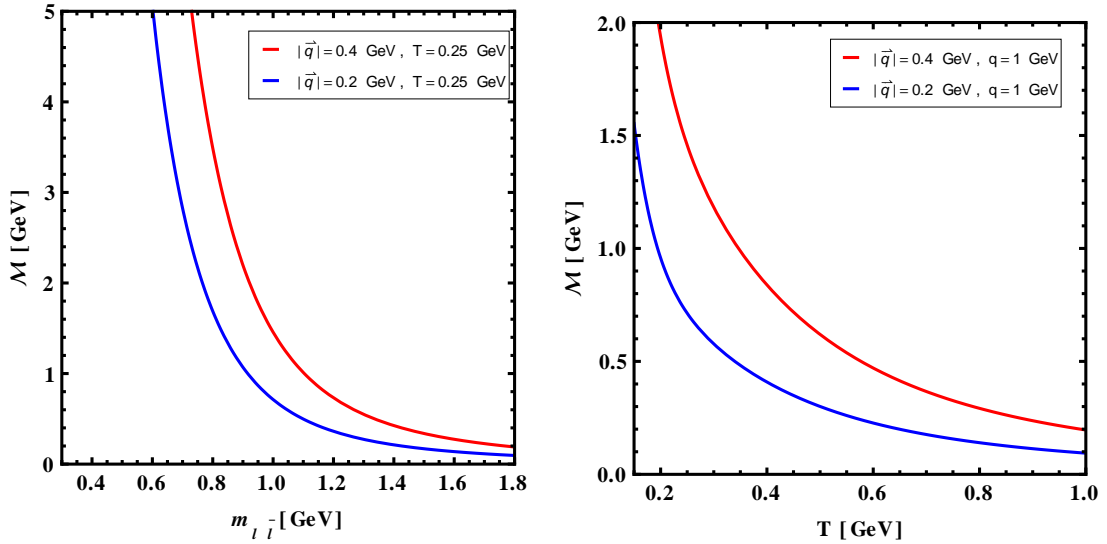


FIG. 4: Variation of \mathcal{M} as a function of invariant mass $m_{l\bar{l}}$ (left) and temperature T (right).

Fig. 5 shows that magnetic field changes the dilepton rate only at lower q^2 , reflecting the behavior of the first and second term of \mathcal{M} (Eq.(IV.16)) as $1/q^2$ and $1/q^4$ at low q^2 . We also note that earlier theoretical calculations without magnetic field disagree with experiment at low

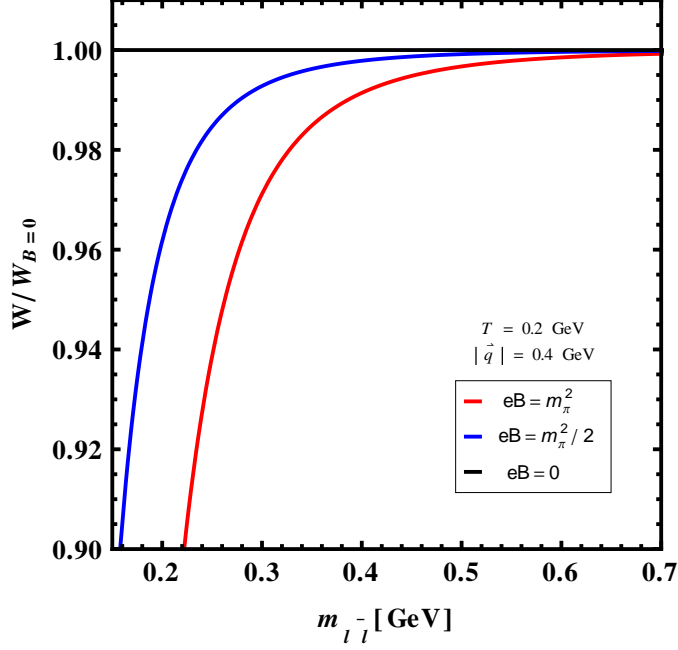


FIG. 5: Plot of the ratio of the dilepton rate with and without the presence of the magnetic field as a function of $m_{l\bar{l}}$.

q^2 [38–40]. It would therefore be tempting to speculate if the effect of magnetic field can bring the agreement, at least in part. However, to verify this speculation, we have to improve our calculation in a number of ways. First, we should include the terms in (IV.9) that are left out in our calculation. Then we need to replace the constant magnetic field by one with its magnitude having (adiabatic) time dependence, as realized in non-central collisions. One can also include the first order QCD correction to the current correlation function [41].

The time dependence of the magnetic field, mentioned above has to be included in the space-time evolution of dilepton production, which is needed to determine its spectrum. Without going into the details of this evolution, we may estimate roughly the effect of the time dependence as follows. The magnetic field realized in the core may be approximated as [7, 10, 11]

$$eB(t) = \frac{8\alpha}{\gamma} \frac{Z}{t^2 + (2R/\gamma)^2} \quad (\text{V.1})$$

where α is the fine structure constant ($= 1/137$), Z and R are the atomic number and radius of the colliding nuclei and γ is the Lorentz contraction factor. This expression excludes large magnetic fields generated immediately after collisions. So it may represent the magnetic field during the QGP phase. Consider Au-Au collision at RHIC, for which $Z = 79$, $R = 6.5$ fm and $\gamma = 100$, giving $eB(t=0) = m_\pi^2/15$. However, considering Pb-Pb collision at LHC, where $Z = 82$, $R = 7.1$ fm and $\gamma = 2800$, we get $eB(t=0) = 1.4m_\pi^2$. Concluding, we find that the effect of magnetic field in

dilepton spectrum is confined to low invariant masses.

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